

Menoufia University
Faculty of Engineering, Shebin El-Kom,
Basic Engineering science Department
First Semester Examination, 2016-2017
Date of Exam: 31/12/2016



Subject: Introduction in Mathematical
Physics

Code : BES 522

Year : Postgraduate students

Time Allowed : 3 hours

Total Marks: 100 marks

Answer the following questions

1) Two equal masses are connected by means of a spring and two other identical springs fixed to rigid supports on either side (Fig. 1), permit the masses to jointly undergo simple harmonic motion along a straight line, so that the system corresponds to two coupled oscillations. Assume that $m_1 = m_2 = m$ and the stiffness constant is k for both the oscillators.

(a) Form the differential equations for both the oscillators and solve the coupled equations and obtain the frequencies of oscillations.

(b) Discuss the modes of oscillation and sketch the modes.

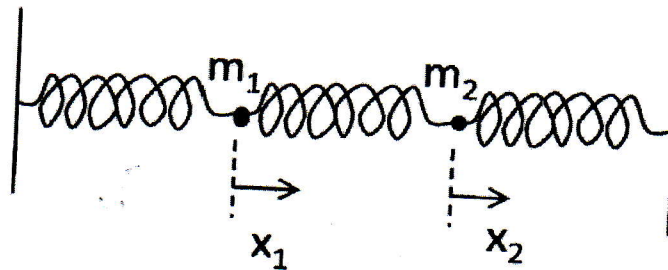


Fig. Coupled oscillator

2) Find the eigen values and eigen functions of the equation:

$$y'' - 4\lambda y' + 4\lambda^2 y = 0, y(0) = 0, y(1) + y'(1) = 0$$

3) Find the general solution of Bessel's differential equation

$$z^2 Y'' + z Y' + (z^2 - n^2) Y = 0$$

where $n \neq 0, \pm 1, \pm 2, \dots$

4) Determine the singular points of each of the following differential equation and specify whether they are regular or irregular.

(a) $z^2 y'' + zy' + (z^2 - n^2)y = 0$

(b) $(z-1)^4 y'' + 2(z-1)^3 y' + y = 0$

(c) $z^2(1-z)y'' + y' - y = 0$

5) Let $u(x,t)$ represent the temperature of a very thin rod of length π , which is placed on the interval $0 \leq x \leq \pi$, at position x and time t . The PDE which governs the heat

distribution is given by $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}$,

where u , x , t and k are given in proper units. We further assume that both ends are insulated; that is, $u(0,t) = u(\pi,t) = 0$ are impose "boundary condition" for $t \geq 0$. Given an initial temperature distribution of $u(x,0) = 2\sin 4x - 11\sin 7x$, for $0 \leq x \leq \pi$, use the technique of separation of variables to find a (non-trivial) solution, $u(x,t)$.

6) Show that the following special functions:

i) $\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin m\pi}$

ii) $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

iii) $\int_0^{\infty} \sqrt{y} e^{-y^3} dy$

iv) $F(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$